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Pion-Condensation Threshold in Nuclear Matter and Thermal Δ-Isobars*

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Abstract: The pion-condensation threshold in symmetric (N = Z) nuclear matter at finite temperatures $0 \leqslant k_B T \leqslant 100$ MeV is calculated. The influence of thermally excited Δ-isobars--present in the nucleon medium--on the threshold is investigated and found to be moderately repulsive (i.e. raising the threshold density).

[Nuclear structure, pion-condensation in nuclear matter, thermal Δ-isobars at finite temperature]

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Since the pioneering work of Migdal[1)] much attention, both in theory and experiment, has been directed to exotic states of baryon matter, such as pion-condensation. Heavy-ion

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collisions at high bombarding energies promise the chance to observe this phenomenon on the earth, whereas conclusions about pion-condensed cores of neutron stars from their cooling are rather indirect and plagued by uncertainties about the equation of state, superfluidity and competing cooling mechanisms.[2)]

In previous calculations concerning pion-condensation, the Δ-isobar occurs either in a Δ-nucleon hole excitation by a virtual pion[3)] or as an admixture to the nucleons, resulting from a full diagonalisation of the baryon system in the presence of a pion-condensate.[4,5)] At finite temperature in nuclear matter, there are already purely thermal baryon excitations present, even in the absence of a pion-condensate. The most important of them, because of its lowest excitation energy, is the Δ-isobar. Consequently, at finite temperature the (negative) pion self-energy Π results in part from the interaction of a pion with a Fermi-sea of Δ-isobars, as indicated in fig. 1.

The threshold of pion-condensation is given by the minimal baryon density ρ_B, required for a solution of the equation

$$\omega^2 = k^2 + m_\pi^2 + \Pi(k,\omega,\rho_B,T) \tag{1}$$

with $\omega = 0$ in spin-isospin symmetric ($N = Z$) nuclear matter. The baryon density ρ_B is the sum of the nucleon density and the Δ-density

$$\rho_B = \rho_N + \rho_\Delta \tag{2}$$

with the Δ-density ρ_Δ given by

$$\rho_\Delta = \rho_B \cdot 4 \exp\left(-\frac{\omega_\Delta}{k_B T}\right)\left[1 + 4\exp\left(-\frac{\omega_\Delta}{k_B T}\right)\right]^{-1} \tag{3}$$

where $\omega_\Delta = 2.3\ m_\pi$ is the mass difference between nucleon and Δ-isobar. The factor 4 results from the four times larger degeneracy of the Δ relative to the nucleon. The pion self-energy consists of an s-wave and a p-wave part, the latter modified by short-range repulsive interactions, incorporated in the Fermi-liquid parameter g':

$$\Pi = \Pi_s + \Pi_p \qquad \Pi_p = \Pi_p^{(o)}[1 - g'k^{-2}\Pi_p^{(o)}]^{-1} \qquad \Pi_s = 0.03\rho\rho_o^{-1}m_\pi^2 \tag{4}$$

with the nuclear matter density $\rho_o = 0.17\ \mathrm{fm}^{-3}$

The zeroth order p-wave self-energy $\Pi_p^{(o)}$ is given by a sum of Lindhard-functions:

$$\Pi_p^{(o)} = -k^2 m_\pi^{-2}[f^2(k^2)U_N + f^{*2}(k^2)(U_{N\Delta} + U_{\Delta N}) + g^{*2}(k^2)U_\Delta] \tag{5}$$

For symmetric nuclear matter the nucleon Lindhard function U_N is given by the principal part integral

$$U_N = 4P\int d^3p(2\pi)^{-3}[n(p) - n(p+k)]\cdot[\varepsilon(p+k) - \varepsilon(p)]^{-1} \tag{6}$$

with the Fermi distribution

$$n(p,T) = [1 + \exp(\varepsilon(p) - \mu(T))/k_B T]^{-1} \tag{7}$$

where $\varepsilon(p) = p^2(2m^*)^{-1}$ is the nucleon energy with an effective mass m^*. The chemical potential $\mu(T)$ is determined from the nucleon density

$$\rho_N = 4\int d^3p(2\pi)^{-3}\ n(p,\mu,T) \tag{8}$$

Furthermore, the Δ-nucleon hole excitation function $U_{N\Delta}$ is given by

$$U_{N\Delta} = \frac{16}{9}\int d^3p(2\pi)^{-3}\ 2n(p)[\varepsilon_\Delta(p + k) - \varepsilon(p)]^{-1} \tag{9}$$

The Δ-single particle energy is $\varepsilon_\Delta(p) = p^2(2m_\Delta)^{-1} + \omega_\Delta$, with $m_\Delta = 9.01\ m_\pi$ and $\omega_\Delta = 2.3\ m_\pi$. The functions $U_{\Delta N}$ and U_Δ have the same structure as their counterparts $U_{N\Delta}$ and U_N; only the roles of nucleon and Δ-isobar are exchanged.

This means the following replacements in the above functions:

$$\begin{array}{ll} n \to n_\Delta & m^* \to m_\Delta \\ \mu \to \mu_\Delta & m_\Delta \to m^* \\ (\mu_\Delta \text{ determined from } \rho_\Delta) & \omega_\Delta \to -\omega_\Delta \end{array} \tag{10}$$

The spin-isospin factor 16/9 in $U_{N\Delta}$ is the same for $U_{\Delta N}$, whereas that for U_Δ is 25 instead of 4 for U_N. In eq. (5), the finite range vertex structure is represented by form factors:

$$\frac{f(k^2)}{f} = \frac{f^*(k^2)}{f^*} = \frac{g^*(k^2)}{g^*} = \frac{\Lambda^2 - m^2}{\Lambda^2 + k^2} \tag{11}$$

with the coupling constants $f^2 = 4\pi\cdot 0.08$ and $f^* = 2f$ and $g^* = 4/5f$ according to the Chew-Low-model.[6)]

In earlier calculations[3,5)] it turned out already that the critical density for pion-condensation is sensitive to the effective nucleon mass m^*, the short-range correlation parameter g', the vertex cutoff Λ, and the temperature k_BT. Measurements and calculations of these quantities are mostly concentrated at nuclear matter density ρ_o with very meager information about the density region above. The cutoff Λ is rather well determined in the range 1-1.4 GeV[7)] and presumably not very sensitive to the density; we adopt the two values 1.2 and 1.4 GeV for our model calculation. The Fermi-liquid parameter g' and the effective mass m^* are not well known at density $\rho > \rho_o$. At $\rho = \rho_o$, g' has values of 0.6-0.7[8)]. In spite of a few model calculations up to $\rho \sim 2\rho_0$, its value remains uncertain above ρ_o. We chose the three values 0.5, 0.6 and 0.7 in order to cope with this uncertainty. For the effective mass m^*, which is probably of all three quantities most sensitive to the density, we make two choices: a density-independent $m^* = 0.8\ m_N$, which is characteristic for ρ_o and a density dependent $m^*(\rho) = m_N(1 - k_F(\rho)k_F(\rho_o)^{-1}\ 0.2)$, which gives $m^* = 0.8\ m_N$ at ρ_o and is an average of several possibilities, offered in ref. 9). The quantity k_F is the Fermi momentum.

The results for the critical baryon density ρ_c are given in fig. 2. At zero temperature, where no Δ-isobars are present, the results are in good agreement with earlier calculations[3,4,5)]. One obvious feature is the rather "parallel" behavior of the phase boundaries with increasing temperature for various cut-offs Λ and parameters g'. Although both quantities have much influence on the critical density, this influence is rather constant between $0 \leqslant k_BT \leqslant 100$ MeV. This is not the case for the two different m^*. At low temperature, $m^*(\rho) < 0.8\ m_N$ gives a smaller pion self-energy and consequently a larger ρ_c. Above $k_BT \sim 50$ MeV another effect gains importance: the "softening" of the Fermi surface by the temperature is relatively smaller for $m^*(\rho)$, because it gives a larger Fermi energy $k_F^2(2m^*(\rho))^{-1}$ than $m^* = 0.8\ m_N$. Both effects tend to cancel with the result that both choices of m^* give nearly identical phase boundaries for $k_BT \gtrsim 50$ MeV.

The Fermi-sea of thermally excited Δ-isobars is of importance only above $k_BT \sim 50$ MeV. Below this temperature, the Δ-density approaches zero exponentially. In eq. (5) the Lindhard-function U_Δ is attractive, whereas $U_{\Delta N}$ is repulsive. The depopulation of the nucleon Fermi-sea in favor of the Δ-isobar is also repulsive, all three together resulting in an increase of the critical density, which is not very dramatic, however; it is a less than 20% effect, even at $k_BT \sim 100$ MeV. The temperature 100 MeV is reached in a typical central heavy-ion collision with bombarding energy $\sim$1 GeV/N, provided complete thermalization is obtained.

We conclude that a Fermi-sea of thermally excited Δ-isobars increases the critical density for pion-condensation in nuclear matter but is below $k_BT \sim 100$ MeV of less importance than the vertex cutoff Λ and the parameter g'.

Of all five considered quantities, Δ-admixture, effective nucleon mass m^*, temperature k_BT, vertex cutoff Λ, and short-range correlation parameter g', the last one still remains the most decisive and unfortunately also (especially at $\rho > \rho_o$) the least well known.

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Figure Captions

Fig. 1. Lowest order p-wave pion self-energy $\Pi_p^{(o)}$.
(a) Direct nucleon Born term and Δ-nucleon hole excitation. (b) Direct Δ Born term and nucleon-Δ hole excitation. Crossed terms omitted for simplicity.

Fig. 2. Critical density $\rho_c(T)$ as function of the temperature and various parameters. The pion-condensed phase is to the right of the curves. Without Δ-Fermi-sea: full curves: Λ = 1.2 GeV and m^* = 0.8; dashed curves: Λ = 1.4 GeV and m^* = 0.8; dotted curves: Λ = 1.2 GeV and $m^* = m(\rho)$. With Δ-Fermi-sea: dash-dotted curves: Λ = 1.2 GeV and m^* = 0.8. Short-range correlation parameter g' as indicated in the figure.

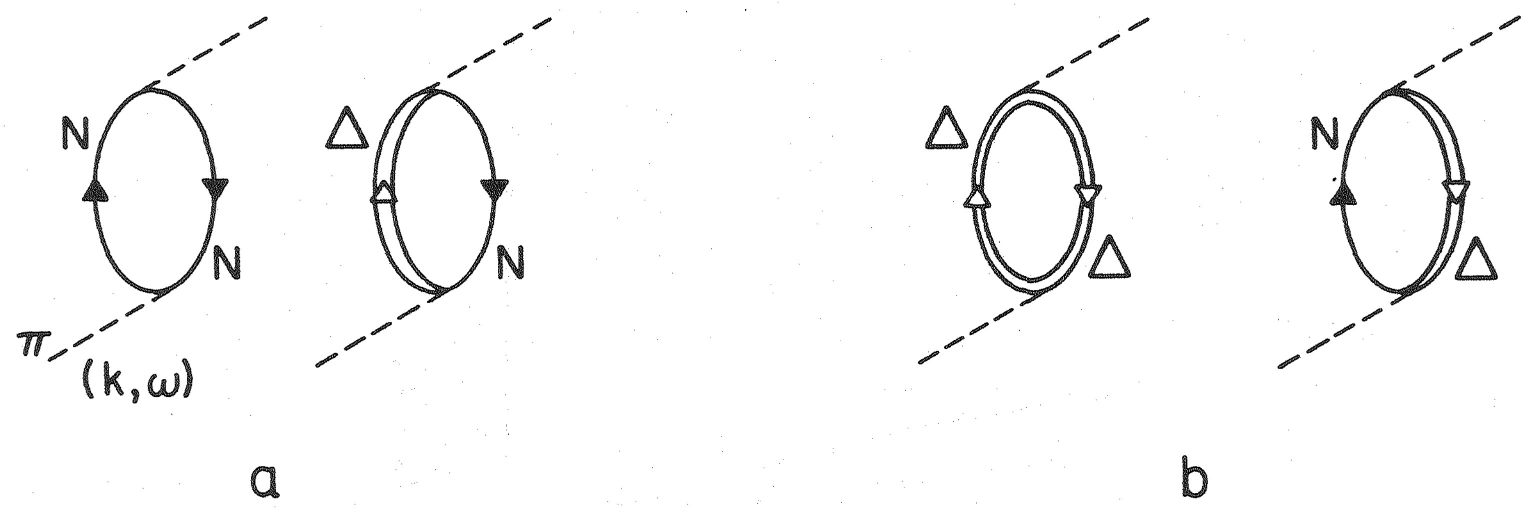

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Fig. 1

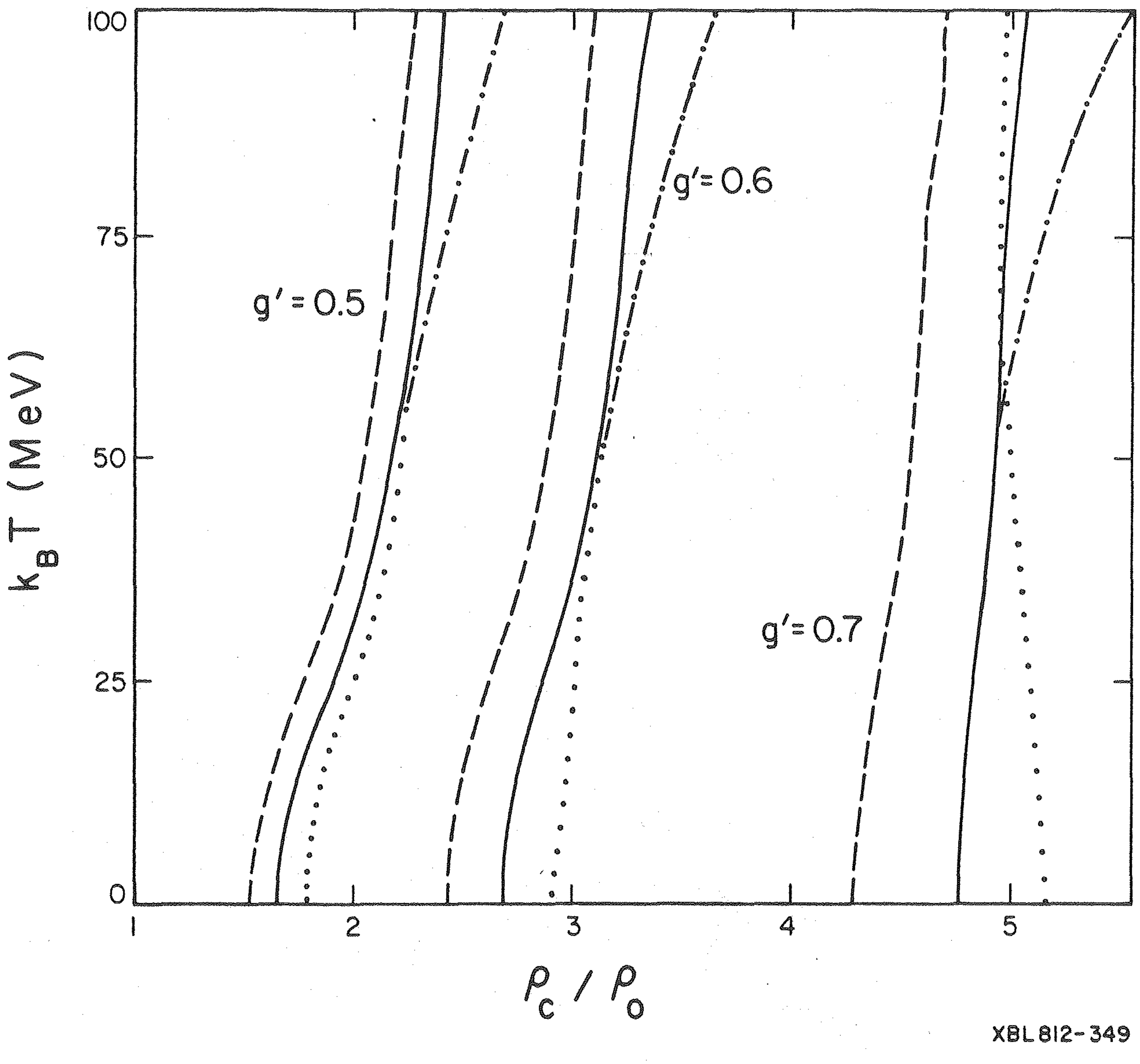

Fig. 2